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ANALOGY BETWEEN TWO MATHEMATICAL MODELS IN THE THEORY OF TWO-PHASE FILTRATION

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It is demonstrated that the general model of nonequilibrium two-phase filtration [1] describes the filtration of a two-phase fluid in the simplest model of a fissured and porous medium.

1. In mathematical models a fissured-porous material is treated as a medium exhibiting dual porosity with each type, i.e., fissured and porous [2, 3], present in each "elementary" macrovolume of the medium. Each of these two distinct types of media (assuming that the other is replaced with a solid skeleton) exhibits a unique porosity and permeability. A fissured-porous medium is the limiting case in which the porosity of the fissures and the permeability of the block tend toward zero.

The combined filtration of two nonmixing incompressible liquids (water and petroleum) in such a medium are described by a system of continuity equations for each phase in the fissures and in the porous blocks as well as in the generalized Darcy's laws for rates of phase filtration in the fissures:

$$M\partial_t s + \nabla \mathbf{u}_1 + q = 0, \quad M\partial_t (1-s) + \nabla \mathbf{u}_2 - q = 0,$$
$$m\partial_t \sigma - q = 0, \quad \mathbf{u}_i = -\frac{k}{\mu_i} f_i(s) \nabla p \ (i = 1,\ 2). \tag{1}$$

Here s and σ denote, respectively, the saturation with water of the fissures and the blocks; q is the volumetric density of the water flow from the fissures to the blocks (i.e., the volume of the water overflowing into a unit volume of the medium per unit time). The direct transfer of liquid to the blocks is not taken into consideration. The fourth continuity equation (for the oil in the blocks) has actually already been accounted for in that the petroleum flow from the blocks to the fissures is assumed to be equal to the water flowing in the opposite direction.

The closing relationship for system (1) must determine the magnitude of the return flows q between the media, which, generally speaking, is a function of the history of the process [i.e., s(t) and σ(t)] at this point. Various simplifying assumptions relative to q were employed in [3-5] which made it possible to write out the closing relationship. It is proposed in [4] to treat q as a universal function (for the given properties of the medium) of the time that the block spends behind the front formed by the water. It was assumed in [5] that q is defined by the instantaneous saturations of the fissures and the blocks at a given point: $q = q(s, \sigma)$. In the following we will deal only with this latter model.

2. In a state of equilibrium between the blocks and the fissures we have $q(s, \sigma) = 0$ and $\sigma = \varphi(s)$ (the function φ is found from the condition of equality for the capillary pressures in the media). For minor deviations from equilibrium, we assume linearization (see Fig. 1):

$$q(s_0,\ \sigma) \simeq q(s_0,\ \sigma_0) + \partial_\sigma q|_{\sigma_0} (\sigma - \sigma_0) = -\partial_\sigma q|_{\varphi(s_0)} (\varphi(s_0) - \sigma).$$

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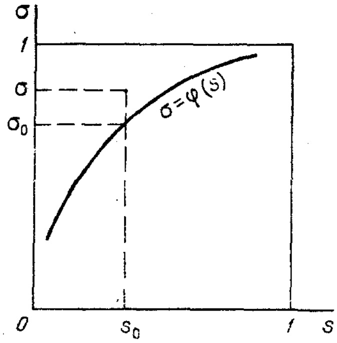

Fig. 1. Function $\sigma = \varphi(s)$ in a state of capillary equilibrium.

Assuming that $-\partial_\sigma q|_{\varphi(s_0)} = K(s_0)$, in accordance with [5] we have

$$q(s, \sigma) \simeq K(s)(\varphi(s) - \sigma). \tag{2}$$

Having substituted (2) into the continuity equation (1) for the blocks, we obtain

$$m\partial_t\sigma = K(s)(\varphi(s) - \sigma). \tag{3}$$

Let us introduce the new variable $\eta = \varphi(s)$ (which, obviously, varies from zero to one). Having denoted $\tau(\eta) = m/K(\varphi^{-1}(\eta))$, where φ^{-1} is the inverse function, we rewrite (3) in the form

$$\eta = \sigma + \tau(\eta)\partial_t\sigma. \tag{4}$$

Let us note that τ is expressed in the dimension of time. System (1), if we eliminate from it the reverse flows q and if we assume the fissure porosity M to be equal to zero, assumes the form

$$m\partial_t\sigma + \nabla \mathbf{u}_1 = 0, \quad m\partial_t\sigma - \nabla \mathbf{u}_2 = 0,$$
$$\mathbf{u}_i = -\frac{k}{\mu_i}\tilde{f}_i(\eta)\nabla p \quad (i = 1, 2), \tag{5}$$

where $\tilde{f}_i(\eta) = f_i(\varphi^{-1}(\eta))$.

Since the fissure volume is negligibly small, the block saturation σ represents, at the same time, the saturation of the fissured-porous medium as a whole. It follows from (4) that when $\partial_t\sigma = 0$, $\eta = \sigma$, so that in steady flow in the macrovolume of the medium we achieve (and this can be measured) the relative phase permeabilities $\tilde{f}_i(\sigma)$ [rather than $f_i(s)$].

From system (4)-(5) we can obtain equations for η and the total filtration rate $\mathbf{v} = \mathbf{u}_1 + \mathbf{u}_2$:

$$\partial_t\eta + \nabla(\mathbf{v}F(\eta)) + \partial_t\tau(\eta)\nabla(\mathbf{v}F(\eta)) = 0, \quad \nabla\mathbf{v} = 0, \tag{6}$$

where

$$F(\eta) = \frac{\tilde{f}_1(\eta)}{\tilde{f}_1(\eta) + (\mu_1/\mu_2)\tilde{f}_2(\eta)}.$$

3. The model based on Eqs. (4)-(5) was proposed by G. I. Barenblatt [1, 3] to describe the nonsteady filtration of a two-phase liquid with consideration of the nonequilibrium distribution of the phases in the microvolume of the medium. This model is based on the assumption that the nonequilibrium phase permeabilities at instantaneous saturation σ are equal to the corresponding equilibrium phase permeabilities for some effective saturation $\eta > \sigma$. The actual and effective saturations at one point are joined together by the simplest of kinetic equations

$$\partial_t\sigma = (\eta - \sigma)/\tau, \tag{7}$$

where τ is the substitution time for the given medium and the pair of liquids.

As was demonstrated earlier, model (4)-(5) is suitable for the description of two-phase filtration in a special kind of medium (a fissured-porous medium). The nonequilibrium in this case is a result of the inhomogeneity of the medium. The effective saturation η (in the terminology of [1]) in this given case, is that water saturation which must prevail in the blocks to attain capillary equilibrium with the fissures (i.e., η is a function of the high-permeability component of saturation for a two-phase medium). The substitution time τ is the time required to establish the equilibrium saturation within the blocks for a fixed density of reverse flow q between the media.

Let us note that system (4)-(5) [unlike the case with (1)] includes only those quantities which pertain to the fissured-porous medium as a whole, rather than only to the separate components of the medium.

4. From (4)-(5) we obtain the equation

$$m\eta + \tau\nabla(\mathbf{v}F(\eta)) = m\sigma, \qquad (8)$$

which relates the functions $\sigma(x)$ and $\eta(x)$ at a single instant of time. According to (8), the value of η, corresponding to an arbitrarily chosen initial distribution of the actual saturation $\sigma(x)$, is established instantaneously, which results in the instantaneous propagation of the perturbations. In the case of a fissured-porous medium, the effect is a consequence of the fact that volume of the fissures has been neglected: when $M \neq 0$, there appears in Eq. (8) a term derived from η with respect to time.

NOTATION

M and m, respectively, the saturations governed by the fissures and the blocks; k, permeability of the fissures; s and σ, respectively, the water saturation of the fissures and the blocks; u, filtration rate; p, pressure; μ, viscosity; f(s), relative phase permeability for a medium made up of fissures; q, volumetric density of the water flowing from the fissures to the blocks; x, radius vector of a point in space; t, time; η, effective water saturation; τ, substitution time; $\tilde{f}(\eta)$, effective relative phase permeability; $F(\eta)$, Buckley—Leverett function; $\varphi(s)$ and K(s), functions determined in place. Subscripts: 1, water; 2, petroleum.

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